Profiles beams optimization problems for remote radiation therapy as multicriterion problem

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Introduction.

Profiles beams optimization problem for tumors in remote radiation therapy has been researched in lot of papers during last years.

This paper deals with formulation of profiles intensity optimization problem as a multi-criterion one and application of finite-size pencil beam algorithm and large-scaled elements method for its numeric solution.

Optimization problem formulation.

Basically, profiles radiation treatment optimization problem belongs to the class of multi-criterion problems as soon as during optimization procedure several risk organs are to be taken into account and radiation treatment quality for each one is determined, for example, by doze mean deviation from zero doze

$$I_k = \frac{1}{n_k} \sum_{i \in V_k} d_i^2, \; k=1,\ldots,m. \qquad (1)$$

At the same time radiation treatment quality for tumor is determined by doze mean deviation from the fixed doze, defined by the patient's clinical tests

$$I_0 = \frac{1}{n_0} \sum_{i \in V_0} (d_i - d_0)^2, \qquad (2)$$

where

d_i- doze in the i-th voxel;

V_k-volume of the k-th risk organ, k=1,…,m;

n_k- number of control voxels of k-th risk organ;

V_0-tumor volume;

n_0- number of tumor voxels;

m-number of risk organs.

Minimization of objective functions I_k, $k=0,1,\dots,m$ combination is the essence of optimization problem. Thus and so we say this problem to be multi-criterion one.

Optimal solution for multi-criterion problem is known to be found out among Pareto solutions in which optimal compromise between concerned criteria is set.

Minimization of linear combinations of initial criteria is one of the ways to find Pareto solutions. That's why we use physical objective functions

$$I=(1-\alpha)\sum_{k=1}^{m}\frac{W_k}{n_k}\sum_{i\in V_k} d_i^2 + \frac{\alpha}{n_0}\sum_{i\in V_0}(d_i-d_0)^2, \qquad (3)$$

where $0<\alpha\leq 1$, $W_k>0$, $\sum_{k=1}^{m} W_k=1$, α - radiation compromise coefficient between tumor and risk organs; W_k – radiation compromise coefficients between risk organs.

The appropriate compromise coefficients meanings are adjusted with account to the maximum admissible meanings for patient's risk organs.

To formulate and solve the initial profile beam fluence optimization problem numerically connection between vector $\psi=(\psi_1,\ldots,\psi_N)^T$ of fluence control parameters ψ_j and doze vector $\mathbf{d}=(d_1,\ldots,d_n)^T$ of control voxels is required

$$\mathbf{D}\psi=\mathbf{d}, \quad (4)$$

where $\mathbf{D}=(D_{ij})$ – (nxN) doze matrix of influence;

D_{ij} – doze in the i-th voxel irradiated by the j-th field segment (the j-th group of pixels);

n- total number of control voxels ($n=n_0+n_1+\ldots+n_m$);

N- number of formed large-scaled elements (groups) determining energy profile beam fluence.

Optimization problem solution method.

Large-scaled elements method gives us an opportunity to decrease number of control parameters essentially by combining small-sized pixels in appropriate large-scaled elements (groups) with uniform radiation intensity control parameter per group. Reduction of control parameters number should not cause the essential loss of accuracy for optimization problem solution.

Various schemes may be applied to form large-scaled elements, for example, conformal mapping of the tumor volume V_0 projection border on the irradiation plane. Rings of conformal mapping of tumor projection border will be large-scaled elements. To form large-scaled elements other schemes also may be used including multi-step processes.

To solve optimization problem vector-matrix representation of objective function is convenient.

$$I(\mathbf{d})=(1-\alpha)\sum_{k=1}^{m}\frac{W_k}{n_k}(P_k\mathbf{d},\mathbf{d})+\frac{\alpha}{n_0}(P_0\mathbf{d},\mathbf{d})-\frac{2\alpha}{n_0}\mathbf{d_0}(\mathbf{p}_0,\mathbf{d}), \qquad (5)$$

where $\mathbf{p_0}=(0,1,\ldots,1,0)^T$- n-dimension vector with the i-th component set to 1 whether the i-th voxel belongs to tumor volume V_0 and to 0 otherwise; $P_0=\mathrm{diag}(\mathbf{p}_0)$- scalar matrix. Similarly vectors $\mathbf{p}_k$ and scalar matrices $P_k=\mathrm{diag}(\mathbf{p}_k)$ corresponding to risk organs are defined.

Substituting doze vector **d** in (5) for radiation fluence vector ψ in (4) we have the following representation of objective function

$$I(\psi)=\frac{1}{2}(C\psi,\psi) - (\mathbf{b},\psi), \qquad (6)$$

where C=(C_{ij})- symmetric positively definite matrix and $\mathbf{b}=(b_1,\ldots,b_n)^T$- vector.

To minimize objective function $I(\psi)$ with limitations $0\leq\psi_j \leq\psi_{\max}$ we use the following iterative process

$$\varphi_j^{(r+1)}=\frac{1}{C_{jj}}(b_j - \sum_{i=1}^{j-1} C_{ij}\psi_i^{(r+1)} - \sum_{i=j+1}^{N} C_{ij}\psi_i^{(r)}), \qquad (7)$$

$$\varphi_j^{(r+1)}=\max\{0,\varphi_j^{(r+1)}), \ \psi_j^{(r+1)}=\min\{\varphi_j^{(r+1)},\psi_{\max}\}, \ j=1,\ldots,N$$

Doze matrix D calculations and doze distributions in voxels are provided with accordance to the pencil beam or the finite-size pencil beam models. These models use energy deposition kernel calculations results for pencil monoenergy sources in bandwidth from 0.1 MeV up to 30.0 MeV.

Conclusions.

Obtained numerical results allow to conclude the following:

1. Iterative method (7) is stable and has a good convergence to optimization problem solution.
2. Calculation method for doze distributions provides sufficient accuracy level and acceptable machine time.
3. Large-scaled elements method allows essentially decrease number of control parameteres for optimization problem without essential loss of accuracy for optimization problem result.
4. Suggested scheme for doze treatment optimization provides the maximum possible uniform doze distribution and required doze levels in tumor and risk organs.

Acknowledgment.

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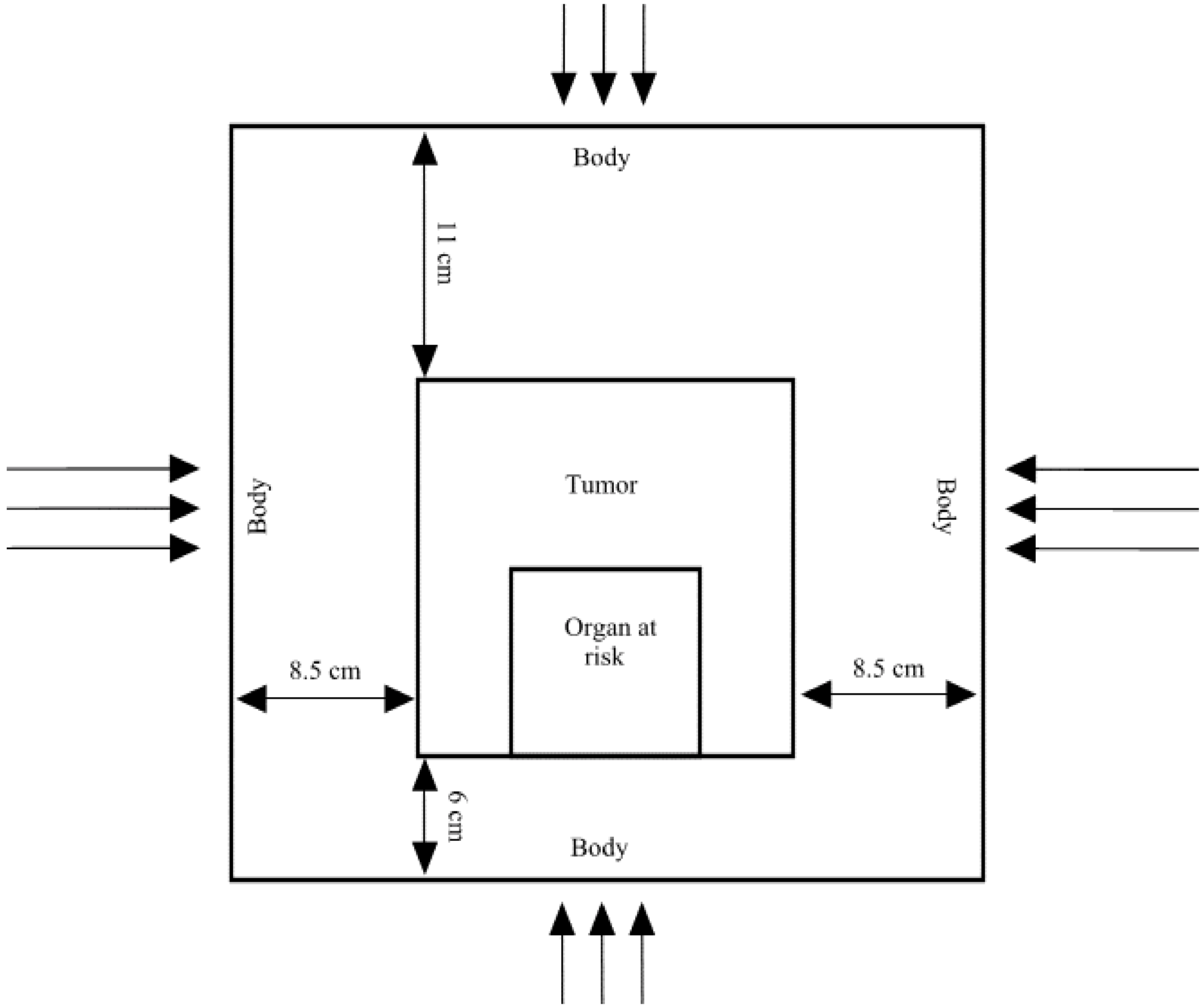
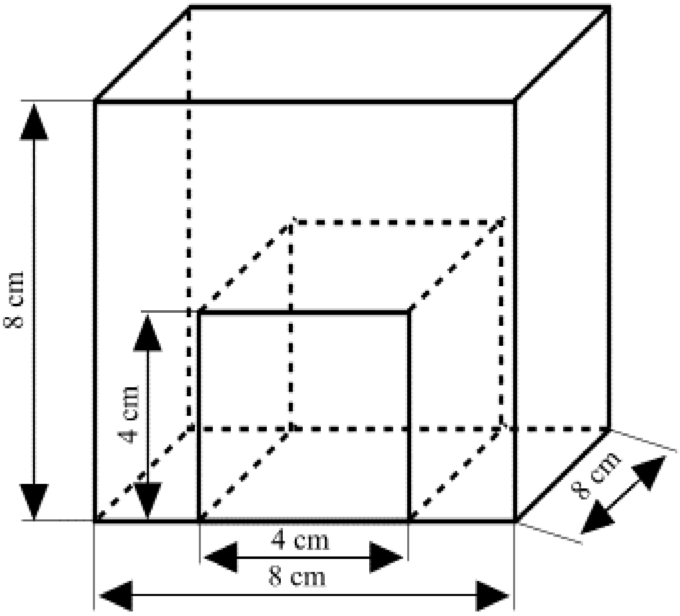
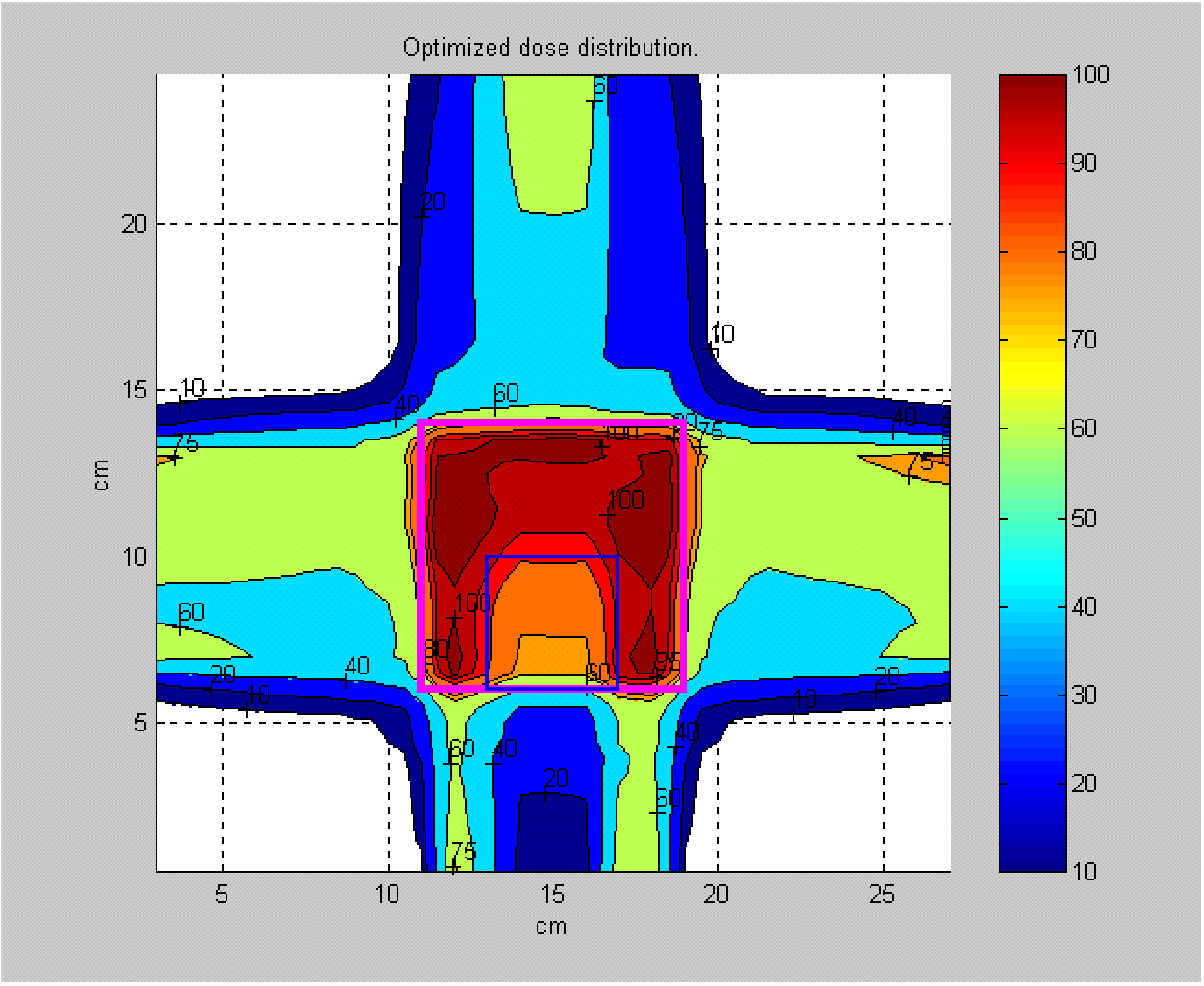
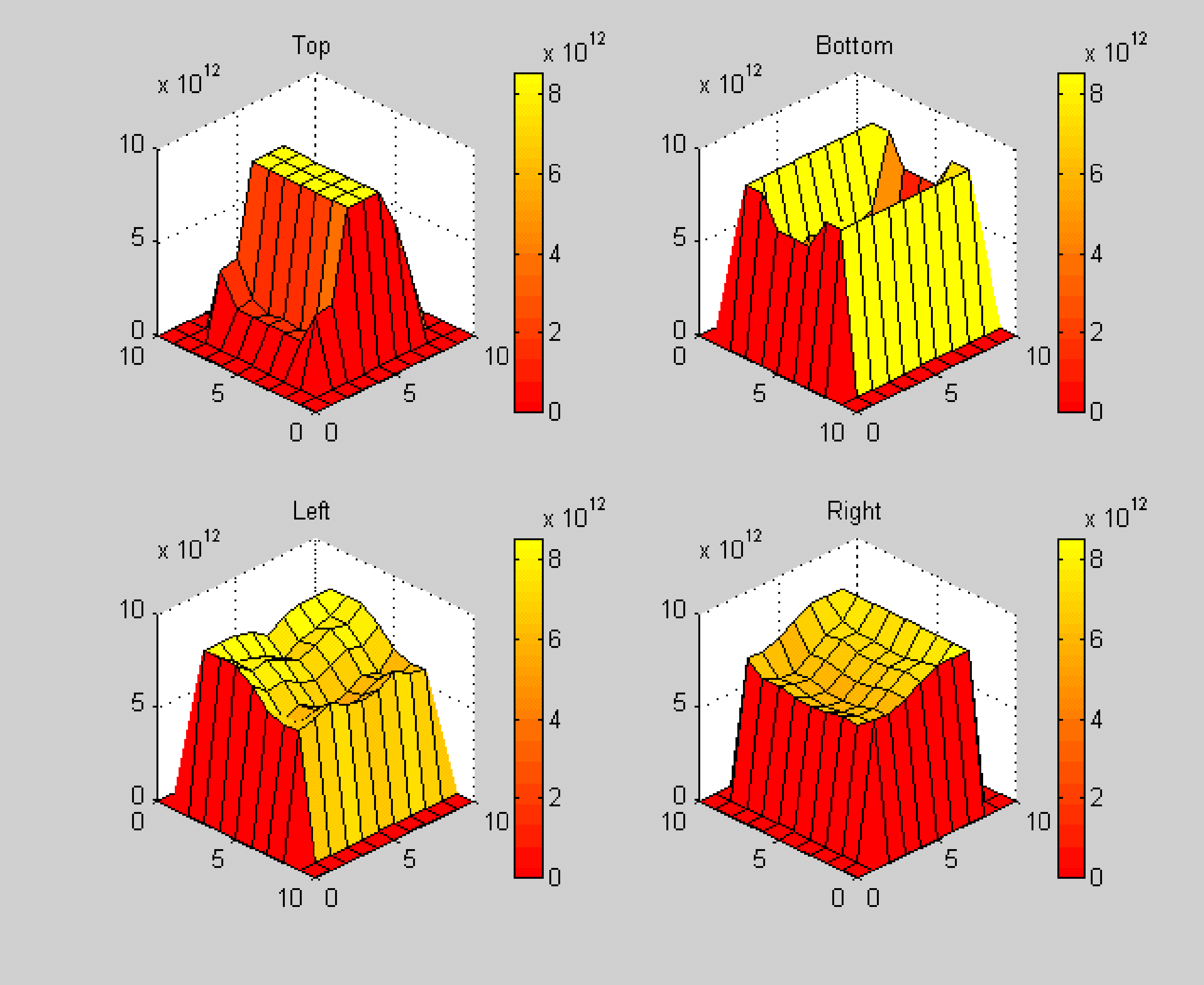




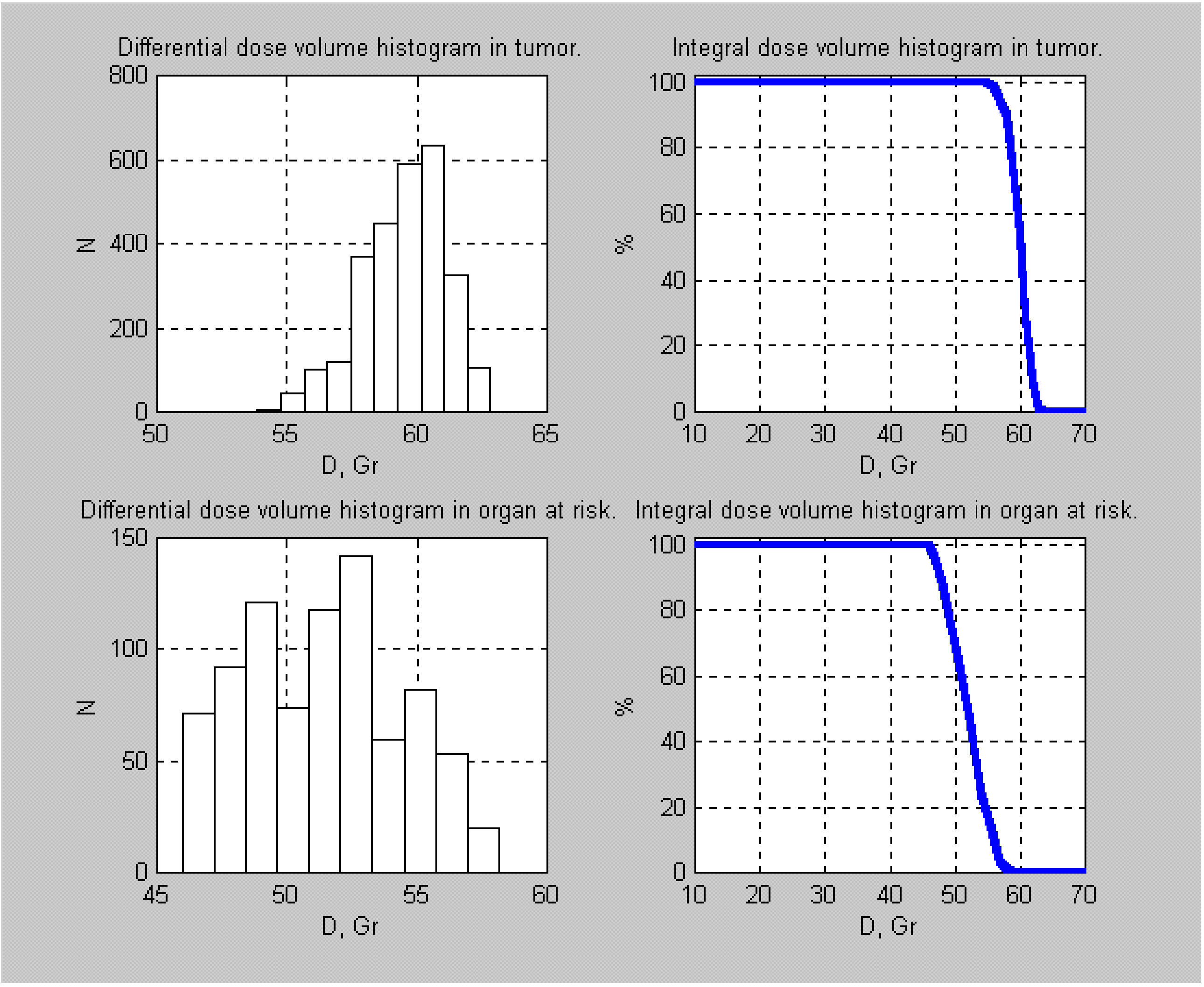
Geometry of the first model problem



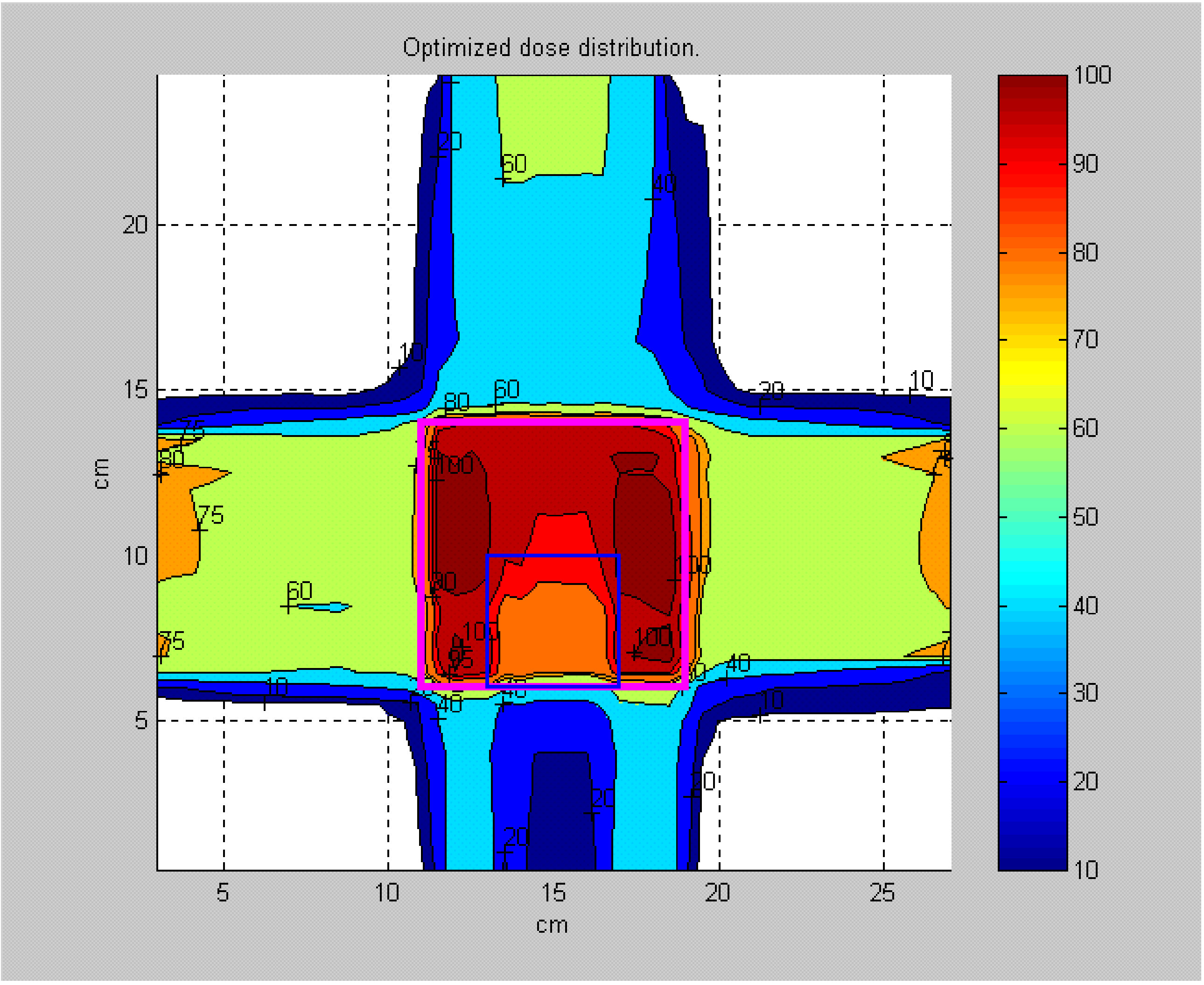
Dose distribution of the first model problem



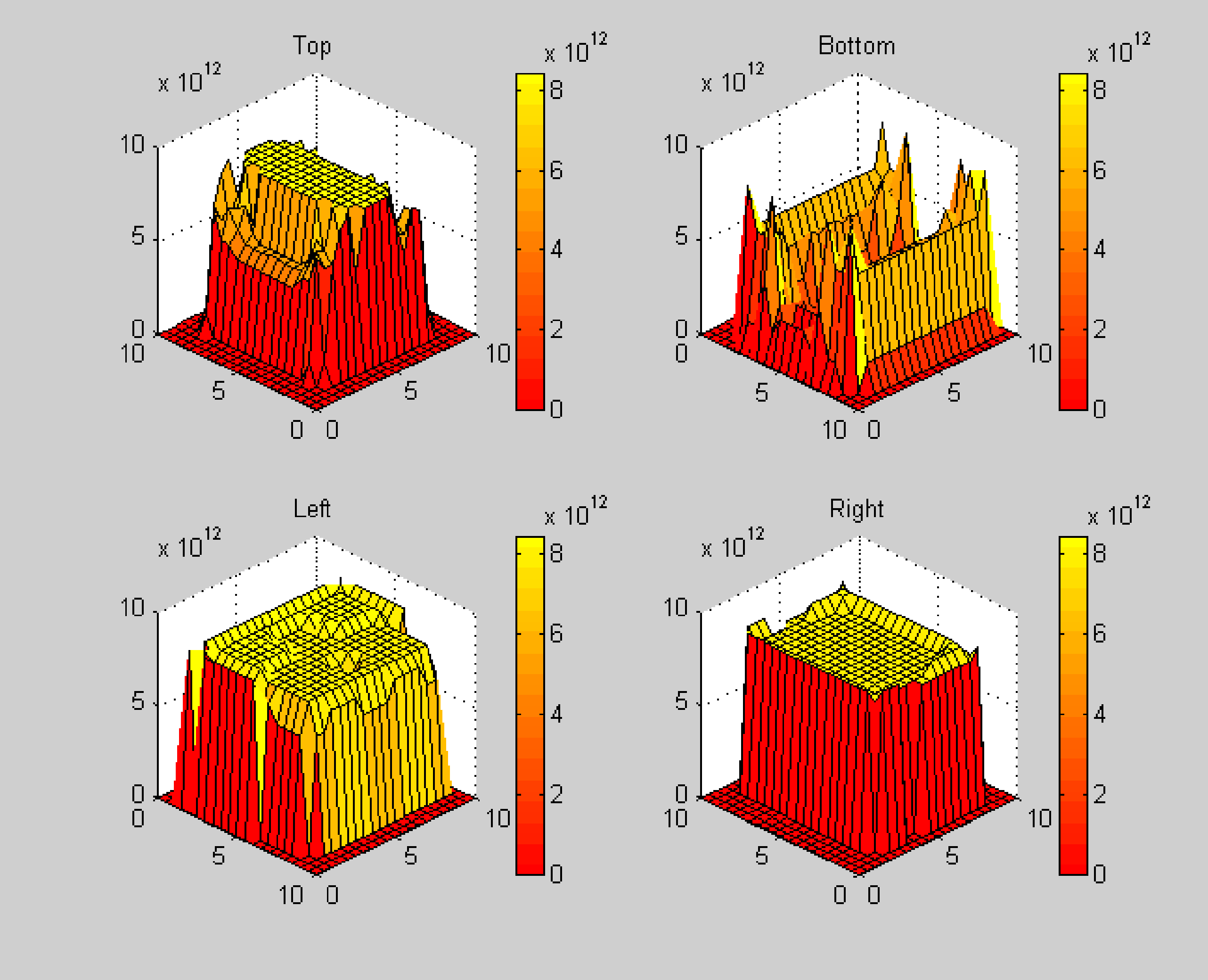
Optimized distribution of primary energy fluence of incident γ-radiation of Co-60



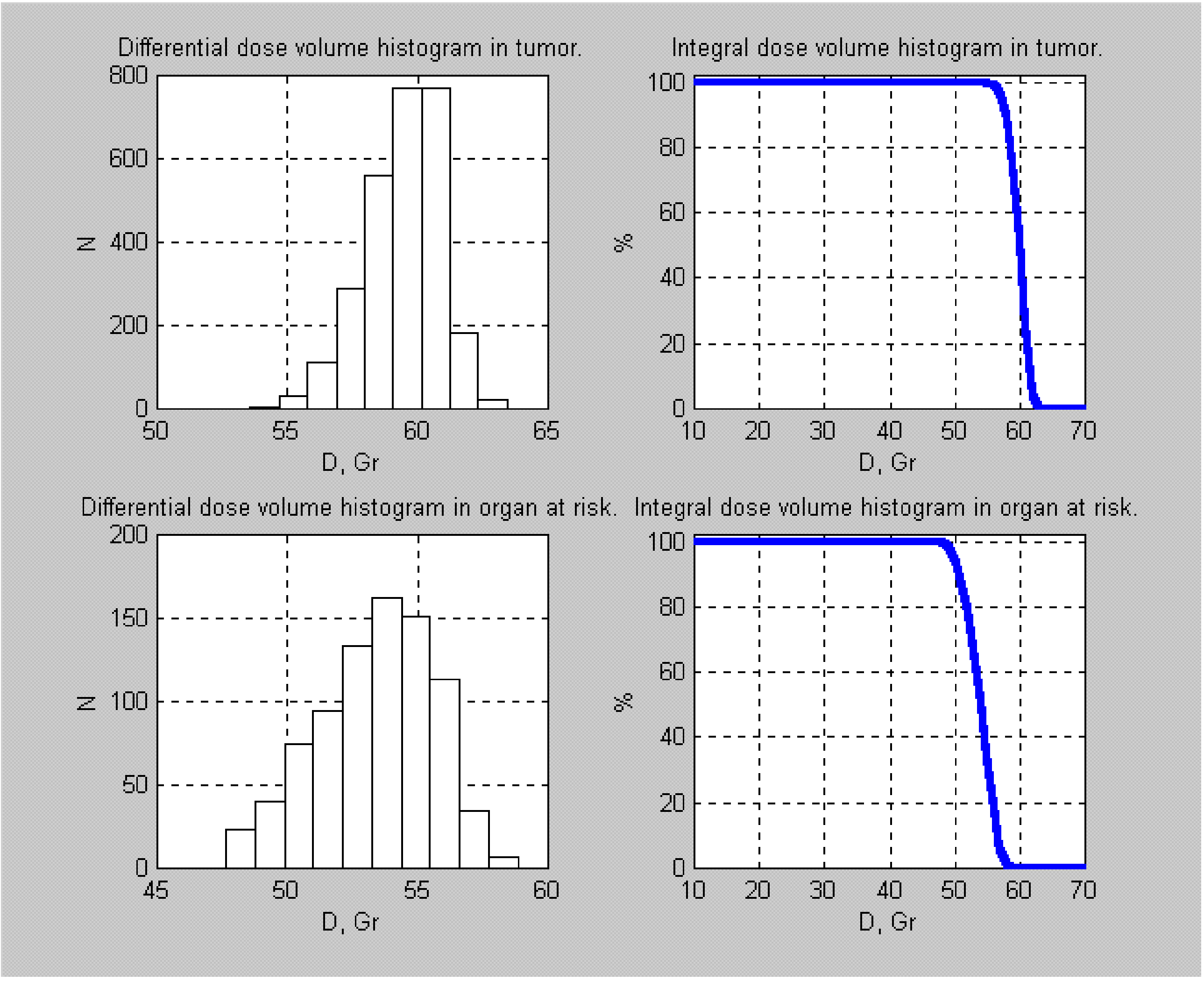
α=0.99
Number of pixels-256
Size of pixel-1 s



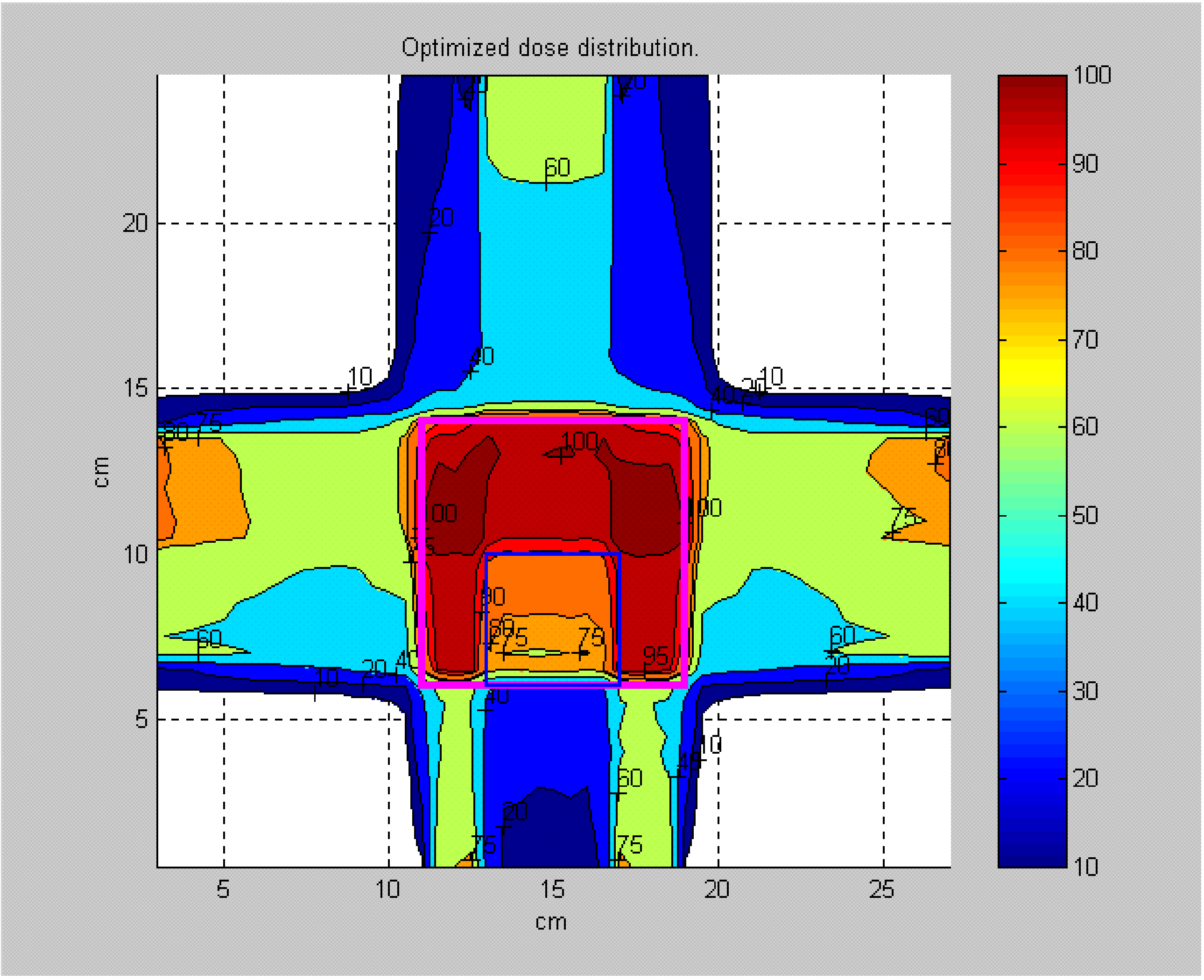
Dose distribution of the first model problem



Optimized distribution of primary energy fluence of incident γ-radiation of Co-60



α=0.99
Formation of groups
number of groups-126
size of pixel-0.5s



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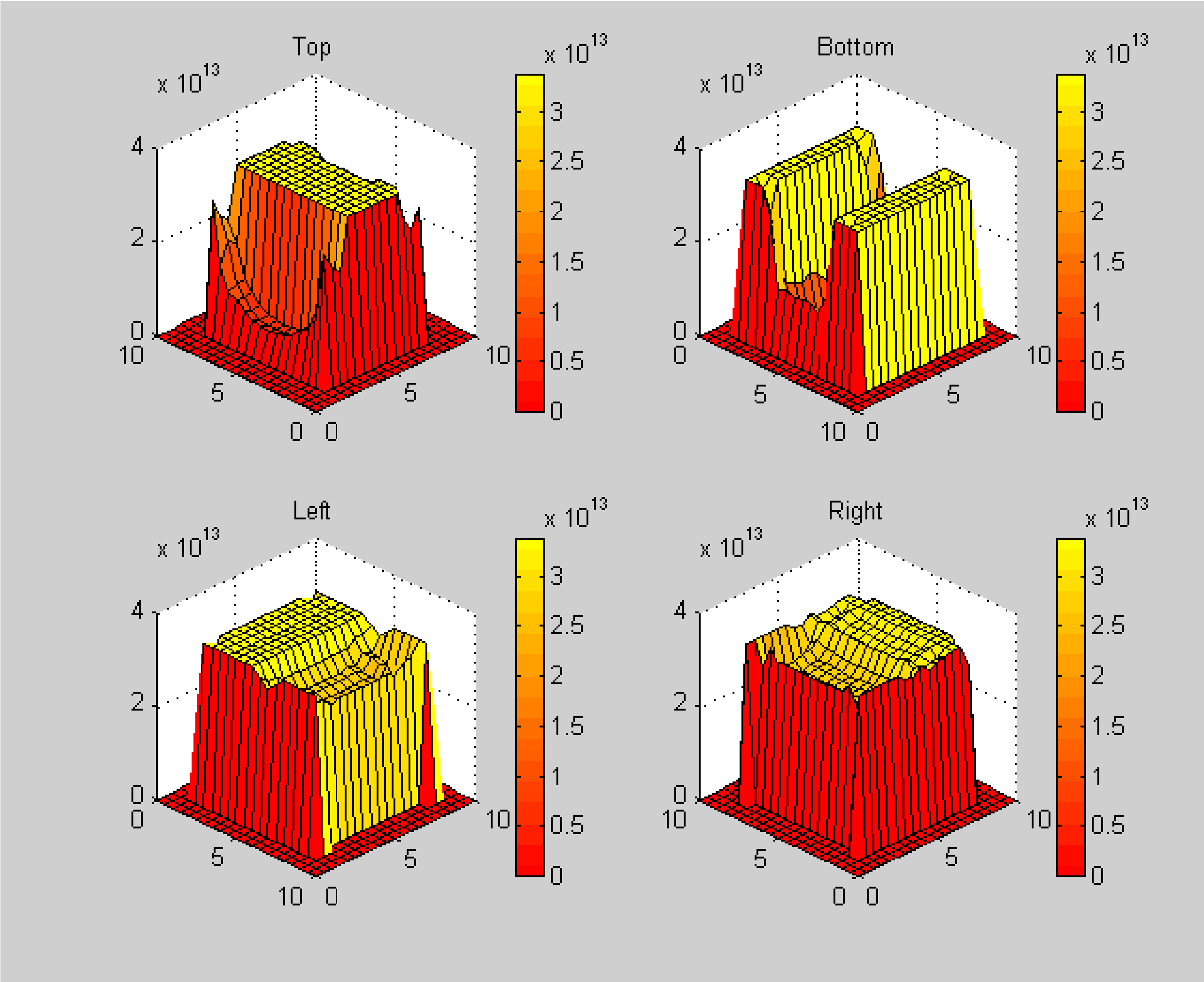
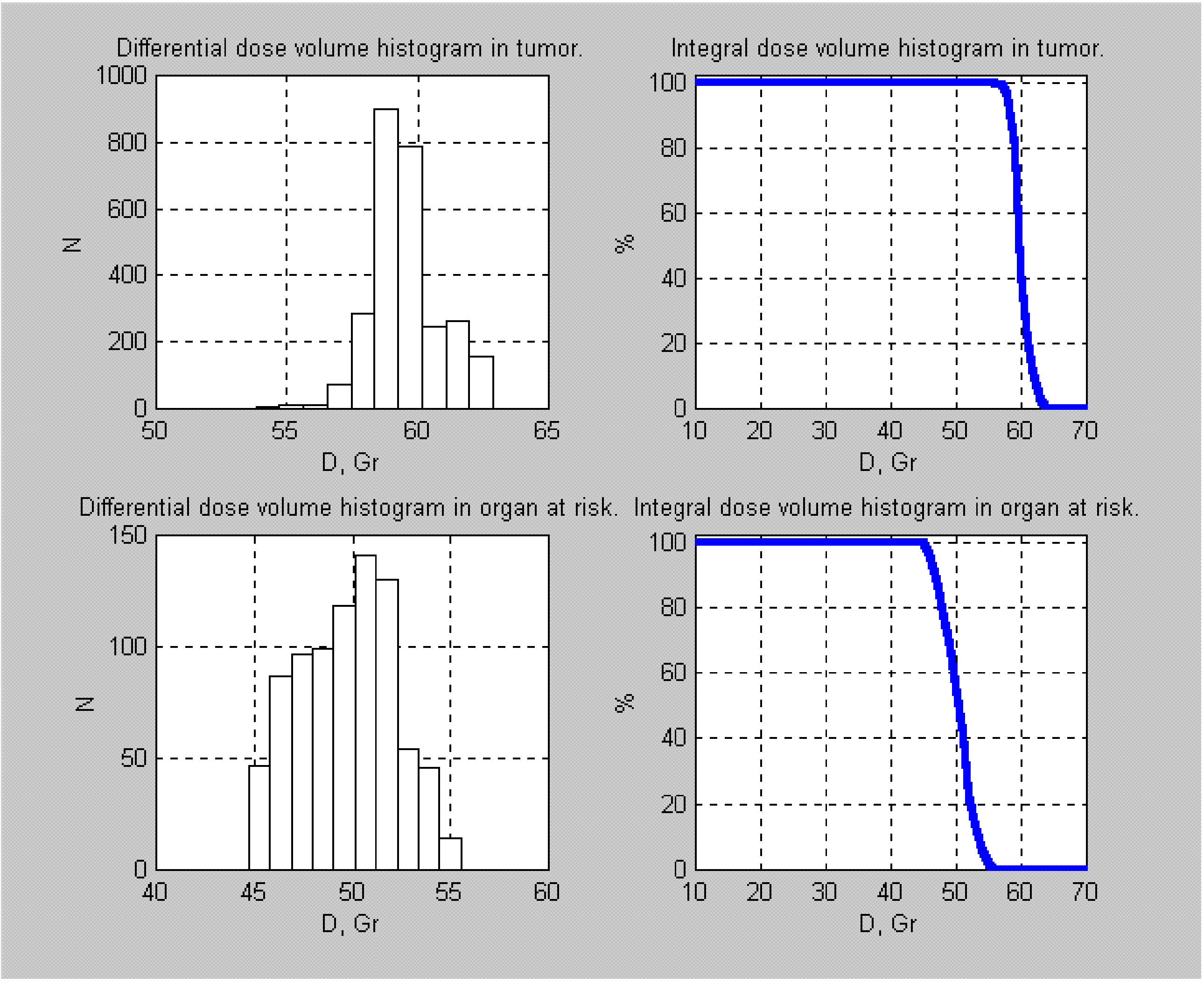
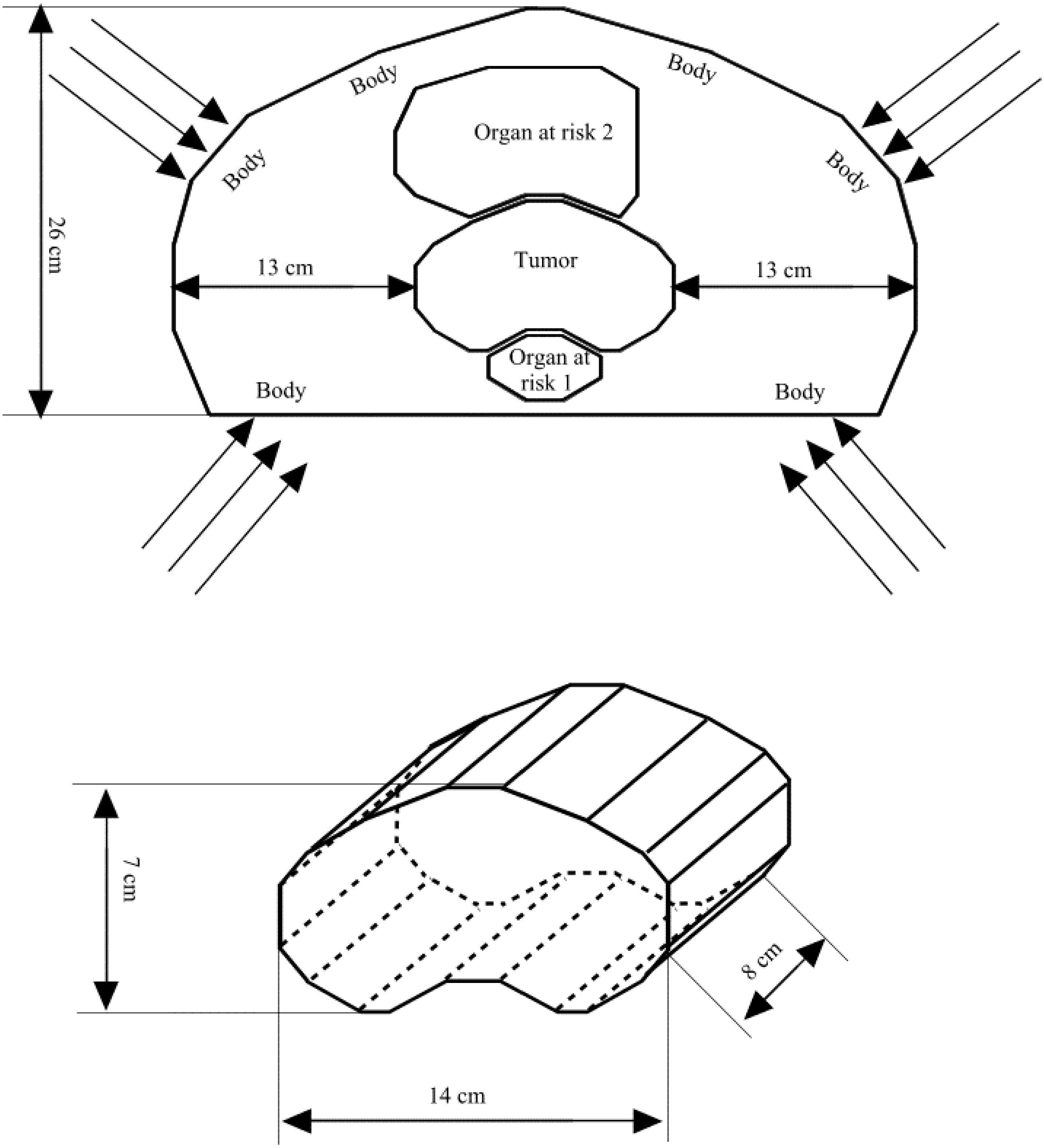


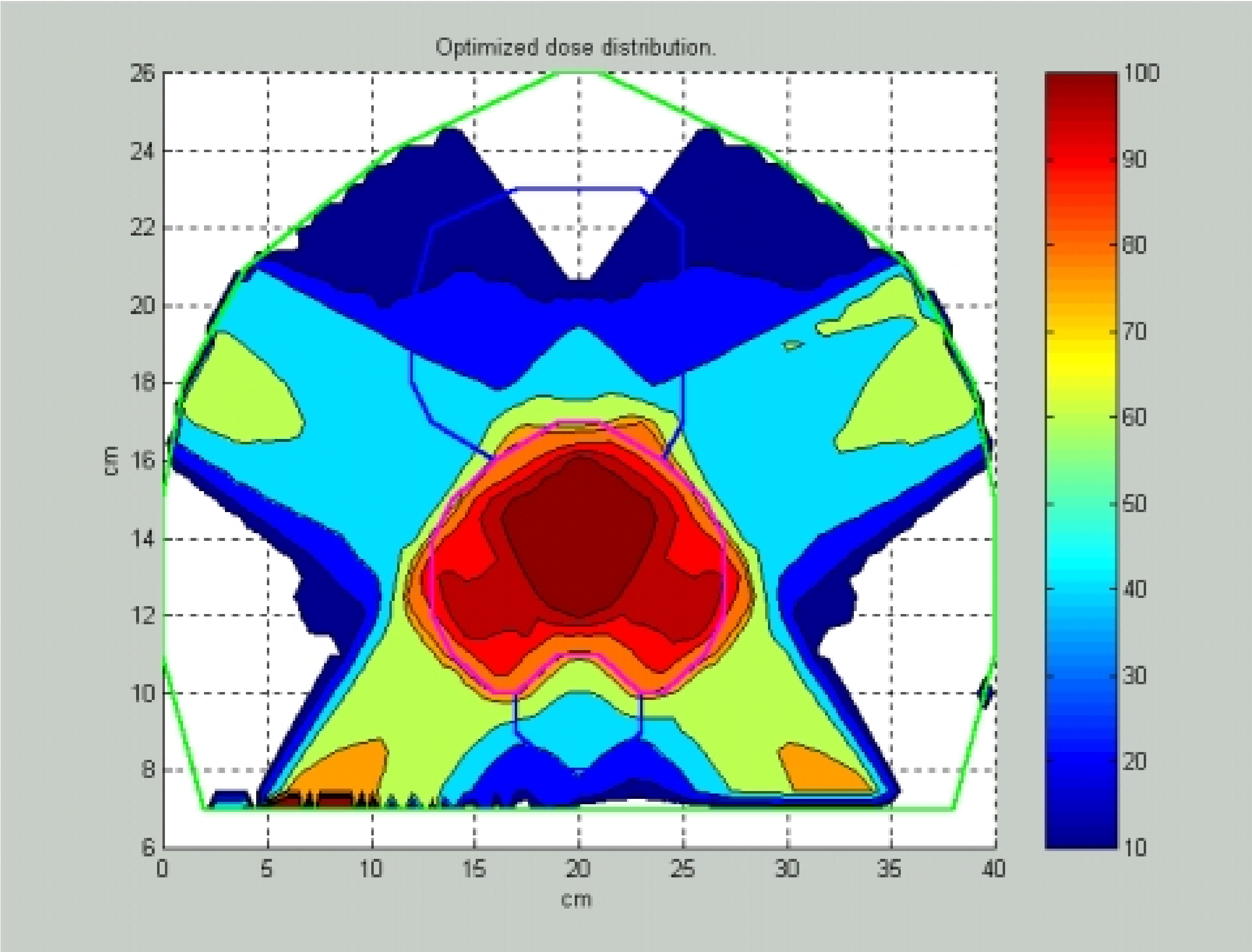
Optimized distribution of primary energy fluence of incident γ-radiation of Co-60



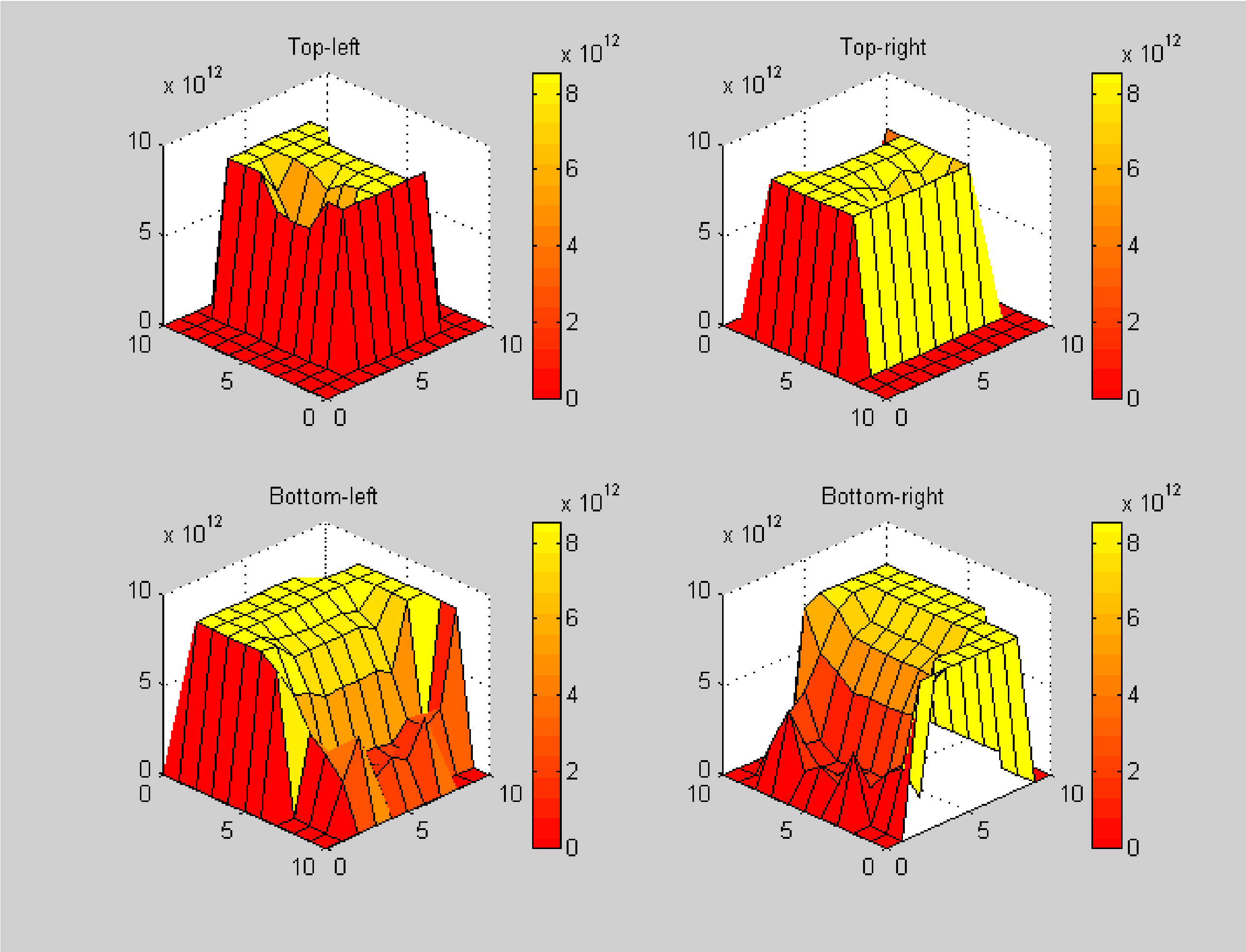
α=0.99
Without formation of groups
number of pixels-1024
size of pixel-0.5s



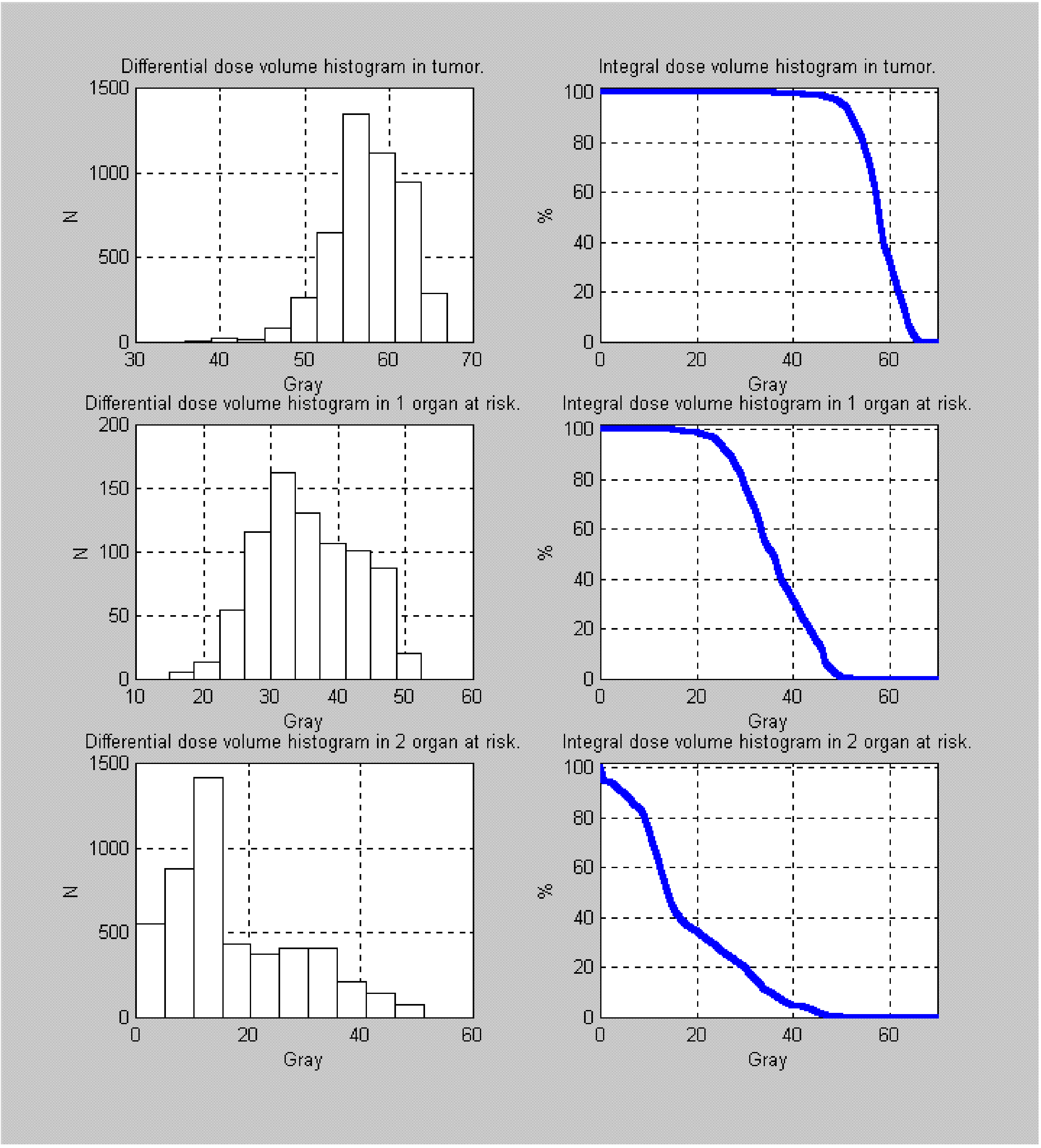
Geometry of the second model problem



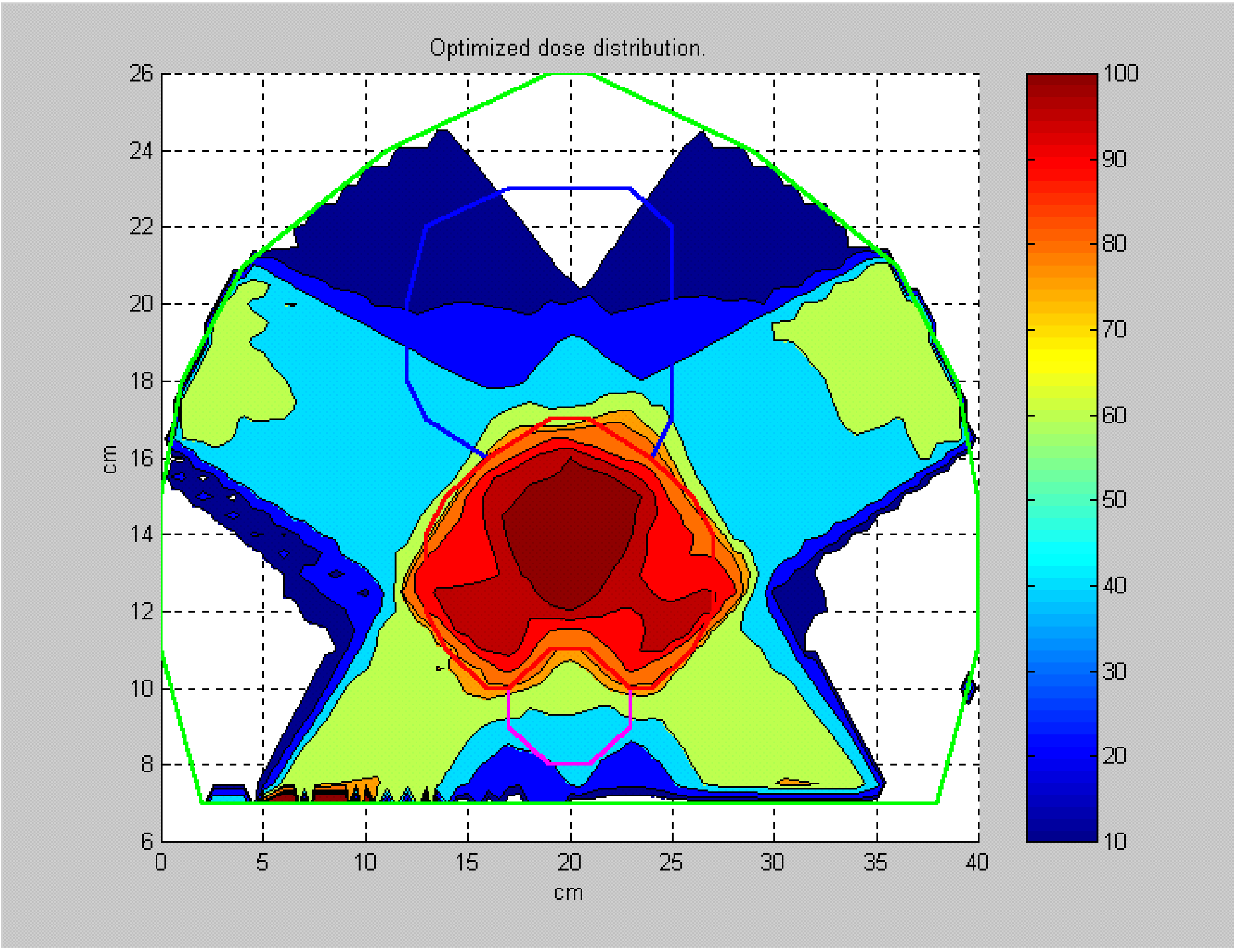
Dose distribution of the second model problem



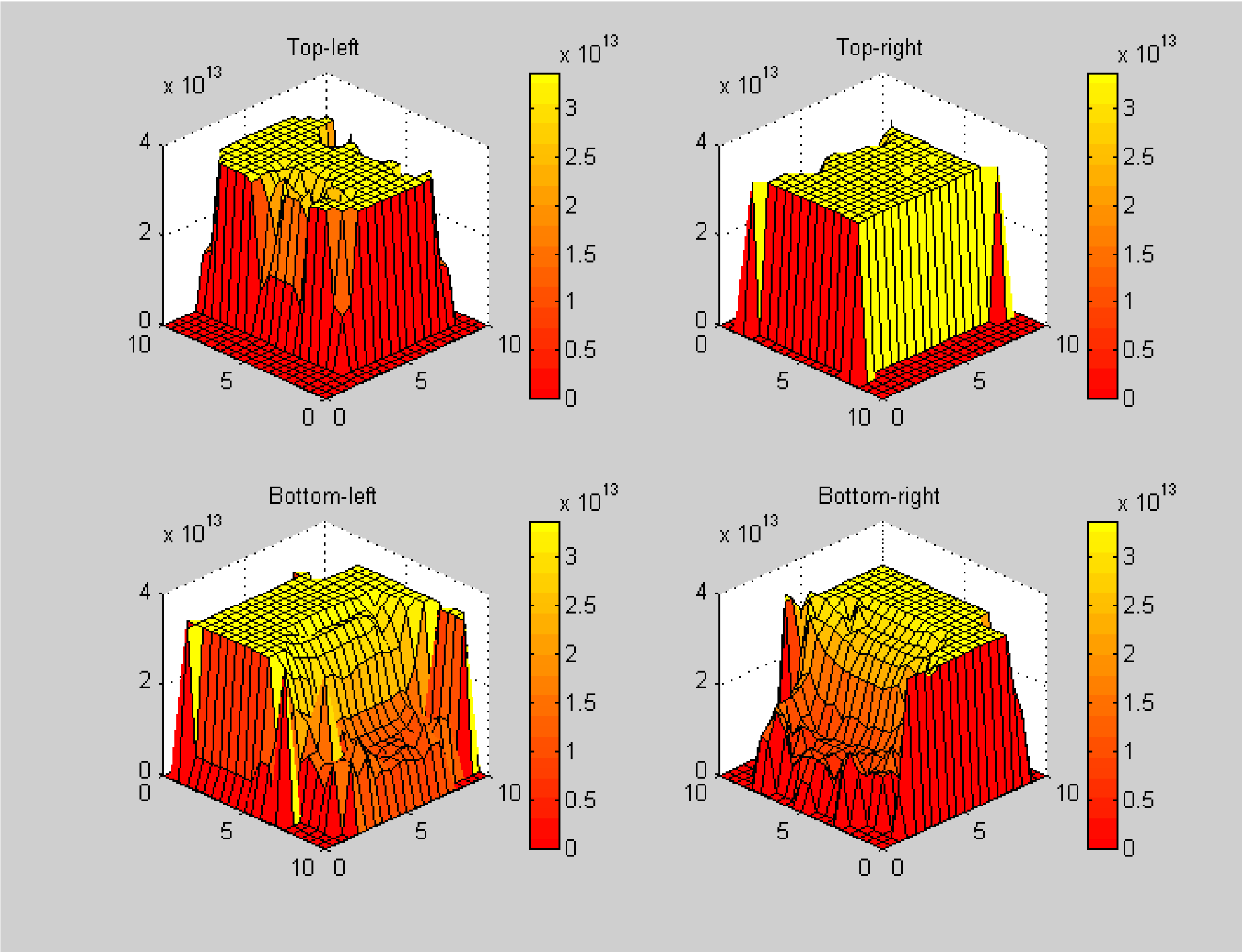
Optimized distribution of primary energy fluence of incident γ-radiation of Co-60



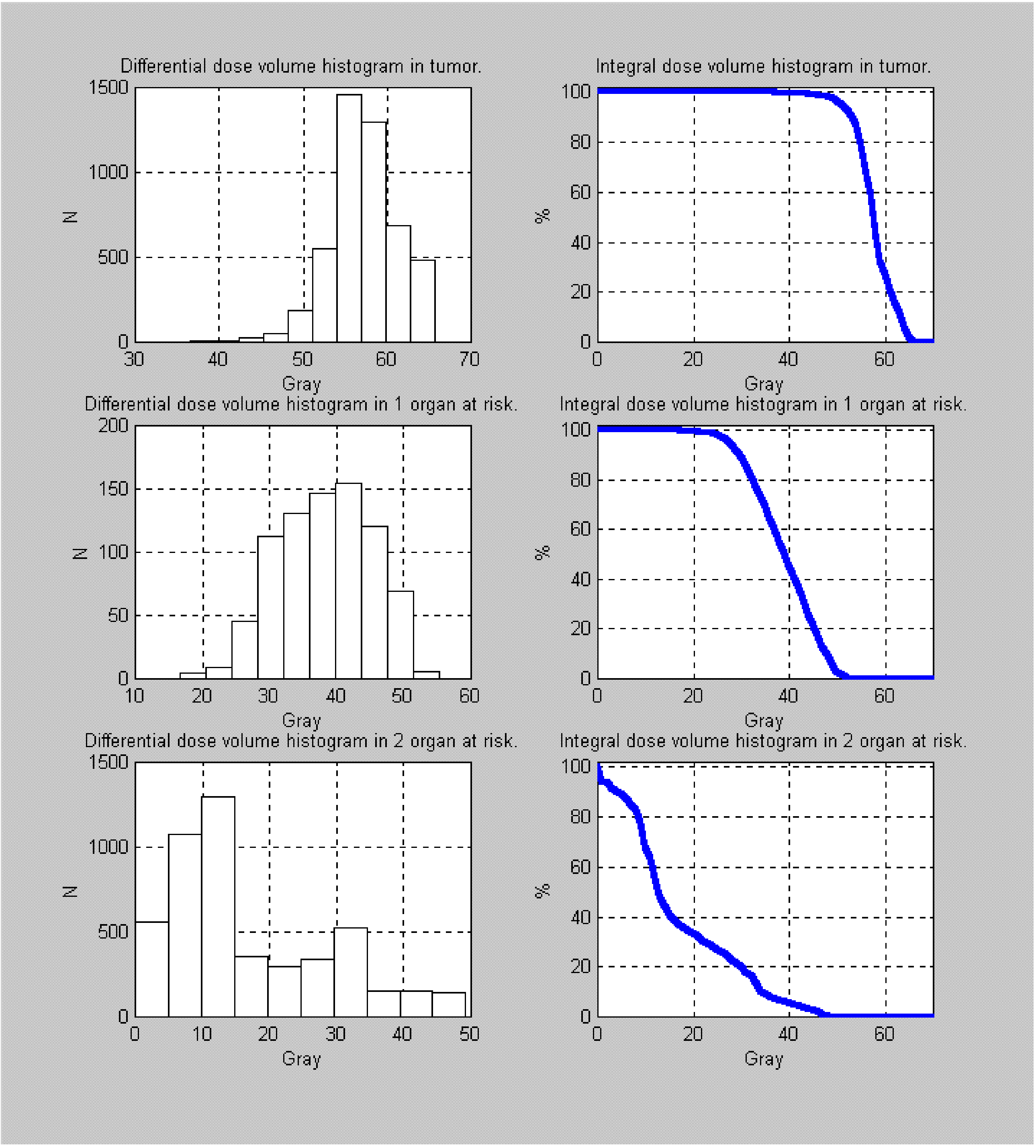
α=0.9
Number of pixels-274
size of pixel-1s



Dose distribution of the second model problem



Optimized distribution of primary energy fluence of incident γ-radiation of Co-60



α=0.9
Formation of groups
number of groups-332
size of pixel-0.5s